

# Theoretical aspects of high- $Q^2$ deep inelastic scattering<sup>‡</sup>

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**Abstract.** We present an overview of the theory of high- $Q^2$  deep inelastic scattering. We focus in particular on the theoretical uncertainties in the predictions for neutral and charged current cross sections obtained by extrapolating from lower  $Q^2$ .

## 1. Introduction

The measurement of deep inelastic scattering cross sections  $d^2\sigma/dxdQ^2$  at high  $Q^2$  provides an incisive test of the Standard Model. Interesting results have already been obtained by the H1 and ZEUS collaborations at HERA — a summary of these can be found in the accompanying experimental review by Mehta [1]. In this talk we will concentrate on some theoretical issues; in particular, how well can we predict cross sections for neutral (NC) and charged current (CC)  $e^-p$  and  $e^+p$  scattering cross sections at high  $Q^2$ , and what can we hope to learn from present and future measurements at HERA? Many of the issues presented here were subsequently discussed in detail by the high- $Q^2$  Working Group at the Workshop [2], and as a result some of the issues raised here were clarified. The importance of the high- $Q^2$  region as a probe of standard and new physics was repeatedly emphasised in the discussions.

We begin by recalling the form of the DIS cross sections in the Standard Model:

$$\frac{d^2\sigma_{NC,CC}(e^\pm p)}{dxdQ^2} = \left\{ \begin{array}{l} \text{standard LO} \\ \text{expressions} \end{array} \right\} + \delta_{\text{QCD}} + \delta_{\text{EW}}. \quad (1)$$

The NC and CC cross sections are obtained from the  $ep \rightarrow eX$  ( $\gamma^*, Z^*$  exchange) and  $ep \rightarrow \nu X$  ( $W^*$  exchange) processes respectively. In (1) the first term on the right-hand side represents the standard leading-order ‘parton-model’ expressions for the deep

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inelastic structure functions ( $F_{2,L,3}$ ). Note that at the high  $Q^2 > \mathcal{O}(10^4 \text{ GeV}^2)$  values measured at HERA, the  $Z^0$ -exchange contribution to the NC structure functions, see below, cannot be neglected. The second term represents perturbative QCD next-to-leading order (NLO) corrections, expressions for which can be found in the literature (see for example Ref. [3]). Away from  $x = 0$  and  $x = 1$  these do not have any particularly dramatic effect on the leading-order cross sections. Since they are generally automatically included in the various computer codes used to fit data and make predictions, they will not be discussed further here. §

The third term on the right-hand side of (1) represents electroweak radiative corrections. These are known to at least  $\mathcal{O}(\alpha)$  and include QED corrections from photon emission off the incoming and outgoing quarks and leptons, and also genuine electroweak corrections from propagator, vertex and box contributions associated with the electroweak gauge boson exchanges. The latter allow a precise theoretical definition of the various electroweak parameters (e.g. vector and axial couplings, gauge boson masses etc.) that appear in the leading-order expressions. For a comprehensive review, see the contribution by Spiesberger in [2].

Three essentially separate types of information can therefore be obtained from high-precision measurements of the cross sections (1) at HERA :

- parton distributions  $f_i(x, Q^2)$  (and  $\alpha_s(Q^2)$ ) at high  $x$  and  $Q^2$ ,
- electroweak parameters, in particular  $M_W$  and  $G_\mu$ , from the space-like  $W$  exchange in the CC cross section,
- limits on, or measurements of, new physics effects (quark substructure, leptoquark production, etc.).

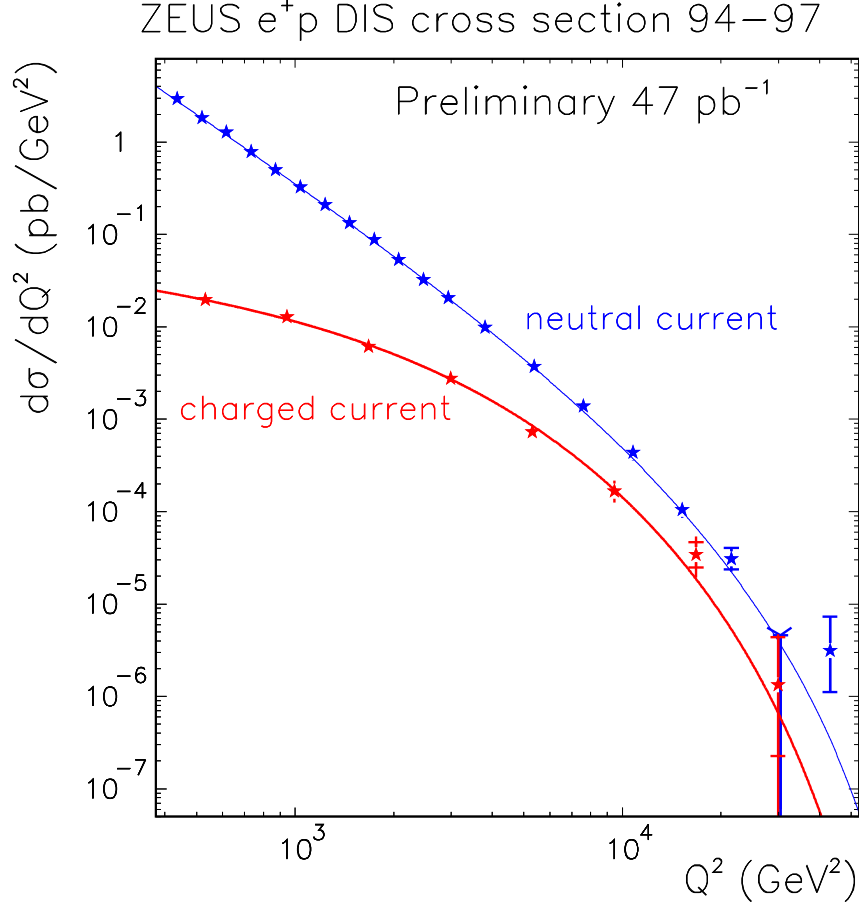
In this talk we will concentrate mainly on the first of these, i.e. the impact of CC and NC measurements on parton distributions and  $\alpha_s$ . Our approach will be to examine the accuracy with which predictions, based on global fits to DIS data at lower  $Q^2$  and NLO QCD evolution to higher  $Q^2$ , can already be made for the kinematic regime covered by HERA. These predictions can then be used as a benchmark to assess the impact of present and future HERA data. Since the issues are slightly different for CC and NC cross sections, we will discuss each of these in turn. Before doing so, for reference we collect together the leading-order expressions for the relevant scattering cross sections: †

- neutral current

$$\frac{d^2\sigma_{NC}(e^\pm p)}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ [1 + (1-y)^2] F_2(x, Q^2) - y^2 F_L(x, Q^2) \right. \\ \left. \mp 2y(1-y)x F_3(x, Q^2) \right] \quad (2)$$

§ Of course going beyond leading order necessitates a choice of renormalisation and factorisation scheme. All quantities referred to in this talk correspond to the  $\overline{\text{MS}}$  scheme.

† In these expressions the proton mass is set to zero, and  $Q^2 = xys$ .



**Figure 1.** Charged and neutral current DIS cross sections at high  $Q^2$ , as measured by the ZEUS collaboration [9] in  $e^+p$  scattering at HERA.

$$\begin{aligned}
 F_2(x, Q^2) &= \sum_q [xq(x, Q^2) + x\bar{q}(x, Q^2)] A_q(Q^2) \\
 xF_3(x, Q^2) &= \sum_q [xq(x, Q^2) - x\bar{q}(x, Q^2)] B_q(Q^2)
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 A_q(Q^2) &= e_q^2 - 2e_q v_e v_q P_Z + (v_e^2 + a_e^2)(v_q^2 + a_q^2) P_Z^2 \\
 B_q(Q^2) &= -2e_q a_e a_q P_Z + 4v_e a_e v_q a_q P_Z^2 \\
 P_Z &= \frac{Q^2}{Q^2 + M_Z^2} \frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha}
 \end{aligned} \tag{4}$$

- charged current

$$\frac{d^2\sigma_{CC}(e^-p)}{dx dQ^2} = [1 - \mathcal{P}_e] \frac{G_\mu^2}{2\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2$$

$$\times \sum_{i,j} \left[ |V_{u_i d_j}|^2 u_i(x, Q^2) + (1-y)^2 |V_{u_j d_i}|^2 \bar{d}_i(x, Q^2) \right] \quad (5)$$

$$\begin{aligned} \frac{d^2 \sigma_{CC}(e^+ p)}{dx dQ^2} &= [1 + \mathcal{P}_e] \frac{G_\mu^2}{2\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \\ &\times \sum_{i,j} \left[ |V_{u_i d_j}|^2 \bar{u}_i(x, Q^2) + (1-y)^2 |V_{u_j d_i}|^2 d_i(x, Q^2) \right] \end{aligned} \quad (6)$$

From these expressions we see that (i) the charged current cross section is relatively suppressed by  $\mathcal{O}(Q^4)$  at small  $Q^2$  where the neutral current cross section is dominated by photon exchange, and (ii) at very high  $Q^2 \gg \mathcal{O}(M_V^2)$ , the charged and neutral cross sections are of the same order. The HERA data confirm this behaviour: Fig. 2 shows the neutral and charged current cross sections, integrated over  $x$ , for  $e^+p$  scattering at high  $Q^2$  measured by ZEUS (see [1]), together with the Standard Model predictions.

## 2. Neutral current cross sections

In QCD, the longitudinal structure function  $F_L$  is suppressed by  $\mathcal{O}(\alpha_s(Q^2))$  compared to  $F_2$  and  $F_3$ , and so at high  $Q^2$  its contribution is numerically small. Ignoring overall factors, we therefore have

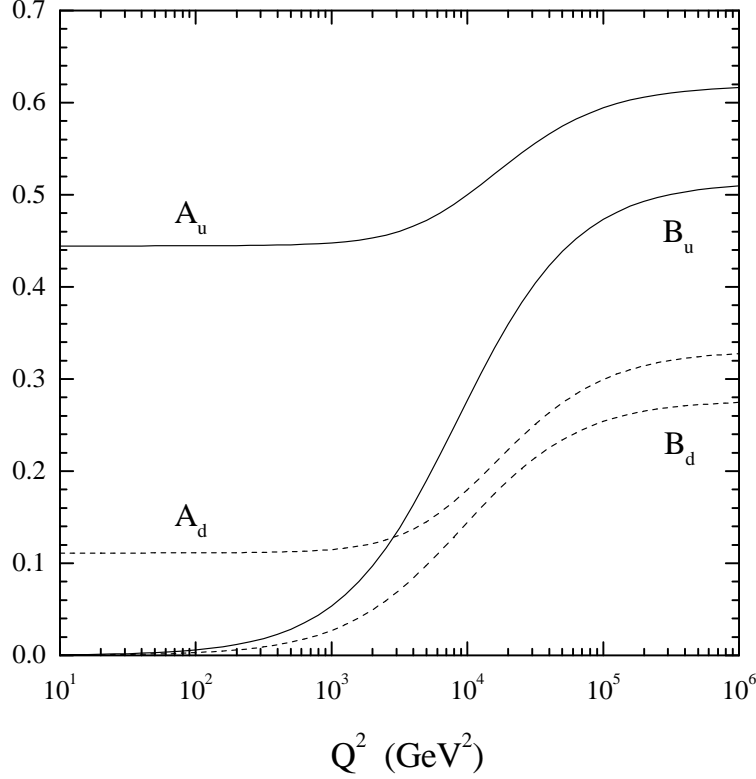
$$\begin{aligned} \sigma_{NC}(e^- p) + \sigma_{NC}(e^+ p) &\sim F_2 = x \sum_q A_q(q + \bar{q}) , \\ \sigma_{NC}(e^- p) - \sigma_{NC}(e^+ p) &\sim x F_3 = x \sum_q B_q(q - \bar{q}) . \end{aligned}$$

The  $Q^2$  dependence of these cross section combinations (disregarding the overall  $1/Q^4$ ) comes from two sources:  $Z^0$  propagator form-factor effects, as contained in the  $A_q$  and  $B_q$ , and logarithmic DGLAP [5] evolution of the parton distributions. Both are visible in current HERA data [1]. Note that as  $Q^2 \rightarrow 0$ ,  $A_q \rightarrow e_q^2$  and  $B_q \rightarrow 0$ . Thus  $F_2$  is the *same* structure function as measured in fixed-target experiments at lower  $Q^2$ . The  $Q^2$  dependence of the  $A_q$  and  $B_q$  for  $u$ - and  $d$ -type quarks is illustrated in Fig. 2. The point to note here is that the relative mix of the two quark types does not change radically as  $Q^2$  increases — up quarks still dominate at high  $Q^2$ . This implies that the uncertainty in the extrapolation of, say,  $F_2^{\mu p}$  from low to high  $Q^2$  at large  $x$  from changes in the relative contributions of the valence  $u$  and  $d$  quarks is very small.

Given a measurement of  $F_2$  at lower  $Q^2$ , how well can we then predict  $F_2$  in the high- $Q^2$  region probed by HERA? Figure 3 is a schematic (i.e. not-to-scale) illustration of the largest sources of uncertainty.<sup>†</sup> First, any measurement error on the low- $Q^2$  data

<sup>†</sup> We are assuming here that the electroweak parameters associated with the  $Z^0$  exchange contribution are already very precisely known from LEP measurements.

## NC pdf coefficients



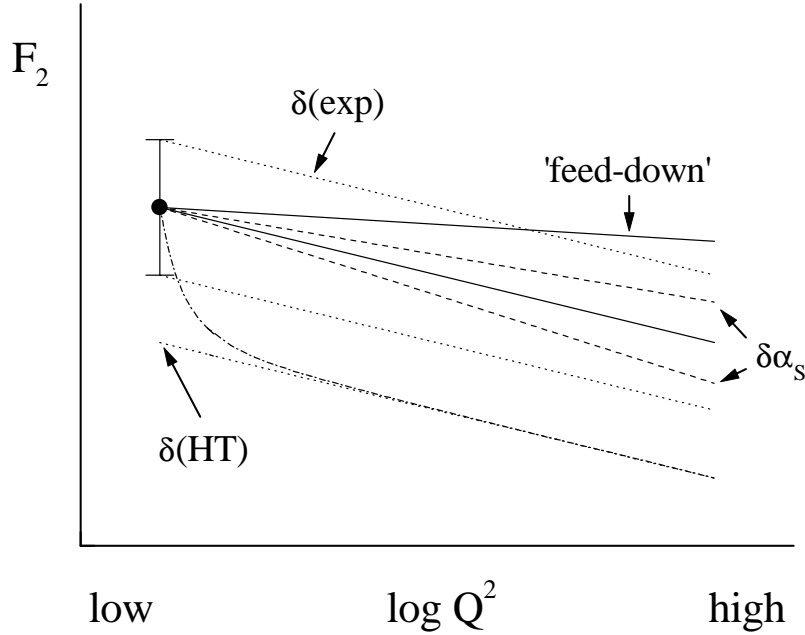
**Figure 2.** The  $Q^2$  dependence of the parton combination functions  $A_q$  and  $B_q$  which appear in the neutral current cross-section expressions of Eq. (3).

propagates directly through to high  $Q^2$ . For fixed-target DIS data at medium-large  $x$ , this uncertainty is of order  $\pm 3\%$  (see for example [4]). Second, any uncertainty on  $\alpha_s(Q^2)$  affects the evolution of  $F_2$  via the large- $x$  DGLAP equation

$$\frac{\partial F_2}{\partial \log Q^2} \sim \alpha_s(Q^2) P^{qq} \otimes F_2. \quad (7)$$

The effect on the evolution of a ‘world average’ value and error,  $\alpha_s(M_Z^2) = 0.1175 \pm 0.005$ , is illustrated in Fig. 4, taken from Ref. [4]. Evidently the error on  $\alpha_s$  induces an uncertainty of order  $\pm 5\%$  in  $F_2$  at high  $Q^2 \sim 10^5 \text{ GeV}^2$ .

An error in the evolution of  $F_2$  could also be made if there is a significant higher-twist contribution to the low- $Q^2$  data that is not taken account in the fitting and subsequent evolution. This is potentially a problem at very large  $x$ , since the higher-twist contributions are expected to behave as  $1/(1-x)Q^2$  relative to the leading-twist contribution. It is difficult to pin down the precise size of this effect — most analyses apply a minimum cut in  $W^2 = (1-x)Q^2/x$  to fixed-target data and fit the remaining

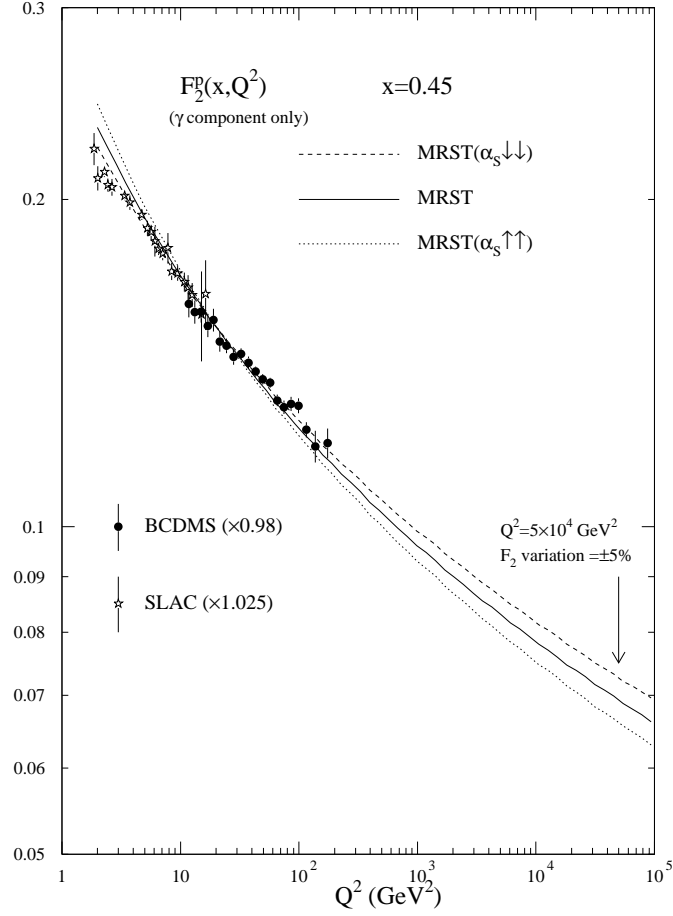


**Figure 3.** Illustration of the different contributions to the uncertainty in the prediction of  $F_2$  at high  $Q^2$ , at a fixed (large) value of  $x$ , given a measurement at lower  $Q^2$ .

data using leading-twist NLO DGLAP. A recent study to quantify the impact of a possible higher-twist contribution on extracted parton distributions is reported in [6].

Finally, the structure function  $F_2$  in the convolution on the right-hand side of the DGLAP equation (7) is sampled at  $x' \geq x$ . The evolution is therefore susceptible to the ‘feed-down’ of an anomalously large contribution to  $F_2$  at  $x \approx 1$ . Such a contribution could escape detection by the fixed-target measurements while still influencing the evolution of  $F_2$  to the HERA region, see for example the study of Ref. [7]. Again, it is hard to quantify the maximum effect that such an anomaly could have on  $F_2$  at high  $Q^2$ . Certainly in global fits that adopt the physically reasonable assumption that (leading-twist)  $F_2$  decreases smoothly to zero as  $(1-x)^n$ , with  $n \simeq 3-4$  at low  $Q^2$ , there is no uncertainty in the evolution of  $F_2$  from the large- $x$  ‘unmeasured’ region.

In summary, if higher-twist contributions have been correctly estimated and if there is no anomalous contribution to  $F_2$  at very high  $x$ , then we should be able to predict the high- $Q^2$  neutral current cross sections at HERA to within about  $\pm 5\%$ , with the main uncertainty appearing to come from the error on  $\alpha_s$ . A HERA measurement at this level of precision would therefore provide a powerful check of the theoretical technology based on leading-twist NLO DGLAP evolution. If there is agreement between the low- and high- $Q^2$  data sets, the latter can be incorporated into global fits to help pin down further



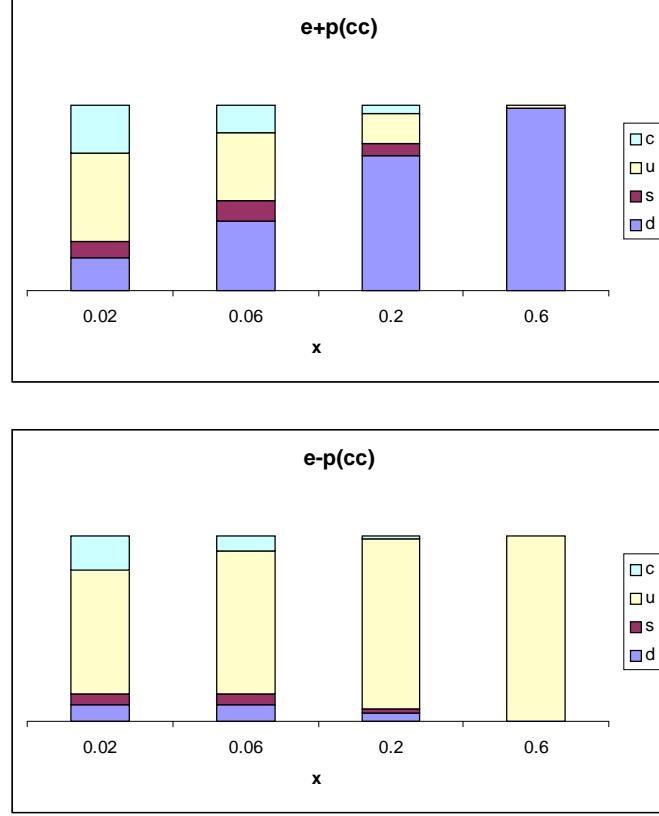
**Figure 4.** The extrapolation of the fits at  $x = 0.45$  to high  $Q^2$  using the MRST, MRST( $\alpha_s \uparrow\uparrow$ ) and MRST( $\alpha_s \downarrow\downarrow$ ) sets of partons, from Ref. [4].

the parton distributions and  $\alpha_s$ . Conversely, any gross deviation from the theoretical predictions could signal new physics.

### 3. Charged current cross sections

The normalisation and  $Q^2$  dependence of the charged current cross sections (5,6) are, in principle, sensitive to the electroweak parameters  $G_\mu$  and  $M_W$ . The current and projected precision on the extraction of these parameters was discussed at some length in the Working Group, see [2]. Notice, however, that there is also potentially useful information on parton distributions, since the flavour decomposition is quite different

**pdf decomposition of  $d\sigma/dx$  ( $Q^2 > 400 \text{ GeV}^2$ )**



**MRST**

**Figure 5.** Parton decomposition of the high- $Q^2$   $e^-p$  and  $e^+p$  CC cross sections.

from that of the neutral current cross sections. Ignoring overall couplings, we have

$$\begin{aligned}\sigma_{CC}(e^+p) &\sim \bar{u} + \bar{c} + (1-y)^2(d+s) \longrightarrow (1-y)^2d, \\ \sigma_{CC}(e^-p) &\sim u + c + (1-y)^2(\bar{d} + \bar{s}) \longrightarrow u,\end{aligned}$$

where the  $x \rightarrow 1$  limit is indicated. The quantitative breakdown is illustrated in Fig. 5, which shows the pdf decomposition of the  $e^+p$  and  $e^-p$  CC cross sections as a function of  $x$  at high  $Q^2$ . Evidently the  $e^-p$  cross section is completely dominated by the  $u$ -quark distribution and, as such, should be predictable with high precision, assuming of course the validity of DGLAP evolution as discussed in the previous section. More interesting is the  $e^+p$  cross section. This is dominated by the  $d$ -quark distribution at large  $x$  (though not to the same extent as the  $u$  distribution dominates the  $e^-p$  cross section). In Fig. 5,



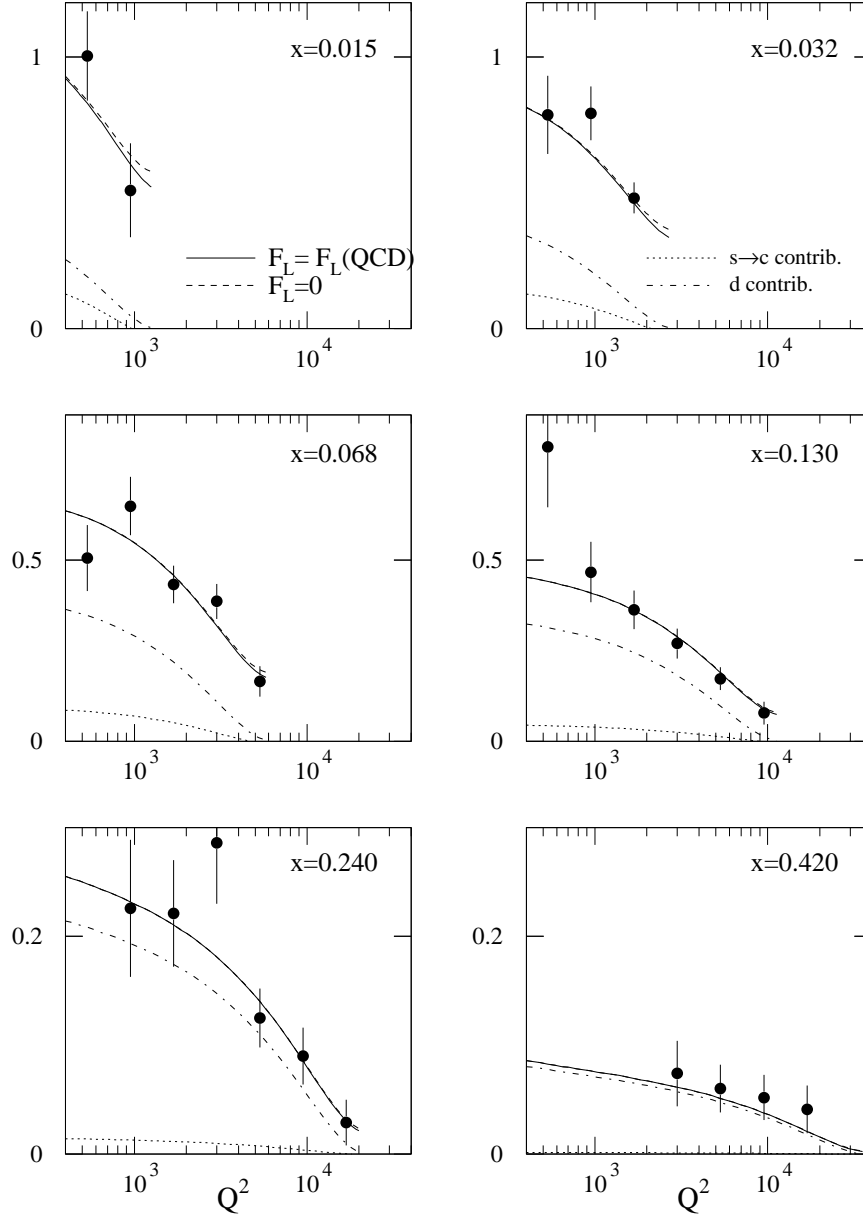
74% and 98% of the leading-order cross section comes from  $e^+d$  scattering at  $x = 0.2$  and  $0.6$  respectively. The ratio  $\sigma_{CC}(e^+p)/\sigma_{CC}(e^-p)$  therefore provides a good measure of the  $d/u$  ratio. Current information on  $d/u$  at large  $x$  comes from fixed target  $F_2^{\mu n}/F_2^{\mu p}$  measurements and the lepton asymmetry in  $p\bar{p} \rightarrow W^\pm + X$ , see for example [4]. In the MRST fit, NMC  $n/p$  data are used to constrain the large- $x$   $d$ -quark pdf in this way. The corresponding predictions for  $\sigma_{CC}(e^+p)$  are compared with the ZEUS data [9] in Fig. 6 [10]. Although the agreement is entirely satisfactory, there is some evidence of a slight excess of data over theory in the largest  $x$  ( $= 0.42$ ) bin. Could this imply that the  $d/u$  ratio is being underestimated in the standard global fits? Any attempt to increase  $d/u$  at large  $x$  in the global fit leads to a direct conflict with the  $n/p$  data. However, Bodek and Yang have argued [8] that the latter should be corrected for nuclear binding effects which, at large  $x$ , lead to a larger  $d/u$  ratio, in ‘better’ agreement with the ZEUS data. This is an issue that deserves more attention, and improved precision on the HERA  $e^+p$  data would be very valuable.

#### 4. Summary

In this brief review we have highlighted some of the physics issues relating to neutral and charged current cross sections at high  $x$  and  $Q^2$  at HERA. Although there is some scope for obtaining information on electroweak parameters, in particular  $M_W$  [2], the main impact of future data is likely to be in testing perturbative QCD evolution via the DGLAP equation and in obtaining information on the pdfs. The  $d$ -quark distribution, for example, is directly probed by the charged current  $e^+p$  cross section. Finally, we note that the HERA high  $x, Q^2$  DIS kinematic region overlaps with the corresponding region that will be probed by many hard scattering processes at the LHC.

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$e^+p$  CC and MRST (ZEUS prelim. data)

**Figure 6.** Comparison of the predictions [10] for charged current  $e^+p$  cross sections using MRST partons [4], with data from the ZEUS collaboration [9].